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Modulational instability of laser radiation in n-GaAs

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Abstract : This paper presents an investigation of a four-wave parametric process when a laser radiation interacts nonlinearly with an ion acoustic wave in a piezoelectric semiconductor, viz., n-type GaAs. The instability results in the modulation of the laser radiation when the drift velocity of electrons due to the laser radiation is comparable to the velocity of the ion acoustic wave in the semiconductor. For typical parameters in n-GaAs at 300°K the growth rate of the low frequency ion acoustic mode is $\sim 10^{1}$ rad. sec^{-1}.

Keywords : Modulational instability, laser radiation, piezoelectric semiconductor, n-type GaAs.

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1. Introduction

The nonlinear interaction of high power electromagnetic waves with ion acoustic modes in a semiconductor plasma has been of considerable interest in recent years (Pandell and Soolloo 1970, Genkin 1975, Pozhela 1981, Salimullah *et al* 1980, Banerjee *et al* 1988 and Guha and Sen 1979). When an ion acoustic wave of angular frequency ω propagates in a plasma, it causes periodic changes in the particle density and temperature at a given point. This results in modulation of the electron density and the electron collision frequency at the point, at the frequency ω. Hence, an unmodulated electromagnetic wave propagating through such a plasma in the direction of the ion acoustic wave becomes modulated at the frequency ω, provided the group velocity of the electromagnetic wave is equal to the phase velocity of the ion acoustic wave.

Guha and Sen (1979) have investigated the modulational instability of a laser beam in a piezoelectric semiconductor, viz., n-InSb, where the nonlinearity arises through the ponderomotive force on electrons only. No such study seems to be reported so far in GaAs in the literature, where the nonlinearity may arise from other sources also. When a high amplitude electromagnetic wave is incident in an n-type GaAs sample, the electrons are heated and are transferred to the high energy low mobility satellite valleys where the effective mass of an electron is higher than that in the central conduction valley. Thus, both the effective mass and the electron collision frequency become functions of the effective electric field intensity. Consequently, in the presence of the high electric field the nonlinearity may arise through the ponderomotive force as well as through the field-dependent average effective mas and electron collision frequency.

In this paper we have studied the four-wave parametric decay of a high power laser radiation into an ion acoustic wave (ω, $\underline{k}$) and two scattered sidebands in a collision-dominated ($\nu \gg \omega$) heavily doped piezoelectric semiconductor, viz., an n-type GaAs sample. We assume that the electron plasma frequency ω_p is nearly equal to the pump wave frequency ω_0 and very large in comparison with the electron collision frequency ν (i.e., $\omega_p \approx \omega_0 \gg \nu \gg \omega$). The growth of the ion acoustic mode whose phase velocity is equal to the group velocity of the incident laser beam causes modulation of the wavefront of the incident laser beam. The dc electric field (bias) is applied to the sample in order to assist the laser beam in transferring electrons from the lower conduction band to the equivalent high energy satellite valley (Seeger 1973).

In Section 2, we have derived the nonlinear dispersion relation of the ion acoustic wave in a homogeneous unmagnetized GaAs. In Section 3, we solve the dispersion relation to obtain an expression for the growth rate of the ion acoustic wave. A brief discussion of the results is given in Section 4.

2. Nonlinear dispersion relation

We consider the propagation of a linearly polarized high power laser radiation in the Z-direction in a homogeneous n-type GaAs sample. The dc electric field $\underline{E}_d$ is applied to the sample in the X-direction.

$$\underline{E}_0 = \hat{x}E_0' \exp\{-i(\omega_0 t - k_0 z)\}, \tag{1}$$

$$\underline{B}_0 = c\underline{k}_0 \times \underline{E}_0/\omega_0, \tag{2}$$

$$k_0 = (\omega_0/c)(\epsilon_L - \omega_p^2/\omega_0^2)^{1/2}, \tag{3}$$

$$\omega_p = (4\pi e^2 n_0^0/m_0)^{1/2} \tag{4}$$

Here, $-e$, m_0, n_0^0, c, ω_0 and ϵ_L are the electronic charge, average effective mass of a free electron, unperturbed equilibrium electron density, the velocity of light in a vacuum, angular frequency of the incident laser radiation and the dielectric constant of the GaAs sample, respectively.

We assume the existence of a low frequency ion acoustic mode (ω, $\underline{k}$) in the semiconductor, propagating in the direction of the incident laser radiation. The ion acoustic wave interacts parametrically with the incident laser radiation and produces two electromagnetic side-bands ($\omega_{1,2}=\omega \mp \omega_0$, $\underline{k}_{1,2}=\underline{k} \mp \underline{k}_0$). Symbolically,

$$(\omega_0, k_0)+(\omega, \underline{k}) \rightarrow (\omega_1, \underline{k}_1)+(\omega_2, \underline{k}_2) \tag{5}$$

The scattered side-bands again interact with the pump wave (ω_0, $\underline{k}_0$) and produce the low frequency ponderomotive force which then drives the low frequency mode (ω, $\underline{k}$) in the semiconductor. If the phase velocity of the growing ion acoustic wave is equal to the group velocity of the incident EM wave, the latter gets modulated by the former. The modulation of the laser radiation depends on the growth rate of the ion acoustic wave.

The response of electrons to this four-wave parametric process is governed by the fluid equations for the electron plasma in the n-GaAs :

$$\frac{\partial n}{\partial t}+\underline{\nabla}.(n\underline{V})=0, \tag{6}$$

$$\frac{\partial \underline{V}}{\partial t}+(\underline{V}\underline{\nabla})\underline{V}=-\frac{e\underline{E}}{m}-\frac{e}{m_0 c}(\underline{V}\times\underline{B})-\nu\underline{V}-\frac{v_e^2}{n}\underline{\nabla}n, \tag{7}$$

where m is the effective mass of electrons, ν is the electron collision frequency, $v_e \simeq (k_B T_e/m)^{1/2}$ is the electron thermal speed. Here, k_B is the Boltzmann constant and T_e is the electron temperature in the n-GaAs sample.

In the presence of the incident laser radiation and the dc electric bias, the linear response of electrons is given by

$$\underline{V}_0=e\underline{E}_0/m_0 i\omega_0, \tag{8}$$

$$\underline{V}_d=-e\underline{E}_d/m_0\nu_0, \tag{9}$$

where ν_0 is the field independent electron collision frequency. The linear response of electrons due to the side-bands is given by

$$\underline{V}_{1,2}=e\underline{E}_{1,2}/m_0 i\omega'_{1,2} \tag{10}$$

Due to the presence of the dc electric field and the electric field of the incident laser radiation the effective mass of an electron and the collision frequency of electrons become functions of the effective field intensity, $E^2_{\rm eff}$ (Guha and Tripathi 1972, Salimullah and Tripathi 1980 and Salimullah *et al* 1981)

$$m=m_0+m_2\left\{\underline{E}_d.\underline{E}+\frac{\nu\omega}{2\omega_0}\left(\frac{\underline{E}_0.\underline{E}_1}{\omega_1}-\frac{\underline{E}_0^*.\underline{E}_2}{\omega_2}\right)\right\}, \tag{11}$$

where

$$m_0 = m(E_{eff}^2), \quad m_2 = \frac{\partial m}{\partial E_{eff}^2}, \tag{12}$$

$$\nu = \nu(E_{eff}^2), \tag{13}$$

$$E_{eff}^2 = E_d^2 + \nu^2 E_0^2/(\omega_0^2 + \nu_0^2) \tag{14}$$

The variation of the average effective mass of an electron, $m(E_{eff}^2)$ and the field dependent electron-phonon collision frequency, $\nu(E_{eff}^2)$ as functions of the effective field intensity can be obtained from the curves of probability distribution function (Conwell 1967) and the variation of transport coefficients (Hartnagel 1969).

We choose the low-frequency ion acoustic mode to be purely electrostatic ($\underline{E} = -\nabla\phi$). We express the various quantities as follows :

$$\begin{aligned}
\underline{E} &= \hat{x}E_d + \underline{E}_0(\omega_0, \underline{k}_0) + \underline{E}(\omega, \underline{k}) + \underline{E}_1(\omega_1, \underline{k}_1) + \underline{E}_2(\omega_2, \underline{k}_2),\\
\underline{B} &= C\underline{k}_0 \times \underline{E}_0/\omega_0 + C\underline{k}_1 \times \underline{E}_1/\omega_1 + C\underline{k}_2 \times \underline{E}_2/\omega_2,\\
\underline{V} &= \underline{V}_d + \underline{V}_0(\omega_0, \underline{k}_0) + \underline{V}(\omega, \underline{k}) + \underline{V}_1(\omega_1, \underline{k}_1) + \underline{V}_2(\omega_2, \underline{k}_2),\\
n &= n_0^0 + n(\omega, \underline{k}) + n_1(\omega_1, \underline{k}_1) + n_2(\omega_2, \underline{k}_2)
\end{aligned} \tag{15}$$

For the highly collisional plasma, we can neglect the pressure gradient term in comparison with the inertial term or collisional force term in the equation of motion of the electrons in the unmagnetized n-GaAs. Thus, taking the nonlinearities arising through the convective term, $\underline{V} \times \underline{B}$ terms and the field-dependent effective mass and collision frequency of electrons, we write the continuity equation and the equation of motion for the plasma electrons due to the low-frequency perturbation mode (ω, $\underline{k}$) as

$$n = n_0^0 \underline{k}.\underline{V}/\omega, \tag{16}$$

$$\begin{aligned}
&\frac{\partial \underline{V}}{\partial t} + (\underline{V}_d.\nabla)\underline{V} + \nu\underline{V}\\
&= -\frac{e}{m_0}\left[\underline{E} + \frac{2m_2}{m_0}\underline{E}_d\left\{\underline{E}_d.\underline{E} + \frac{\nu\omega}{2\omega_0}\left(\frac{\underline{E}_0\underline{E}_1}{\omega_1} - \frac{\underline{E}_0^*.\underline{E}_2}{\omega_2}\right)\right\}\right]\\
&- \frac{e}{2m_0 c}[\underline{V}_0 \times \underline{B}_1 + \underline{V}_1 \times \underline{B}_0 + \underline{V}_0^* \times \underline{B}_2 + \underline{V}_2 \times \underline{B}_0^*]\\
&- \tfrac{1}{2}[(\underline{V}_0\nabla)\underline{V}_1 + (\underline{V}_1\nabla)\underline{V}_0 + (\underline{V}_2.\nabla)\underline{V}_0^* + (\underline{V}_0^*.\nabla)\underline{V}_2],
\end{aligned} \tag{17}$$

where, the asterisk represents complex conjugate of the quantities involved. Considering the Z-component of the ponderomotive force only, one obtains the following nonlinear low-frequency density perturbation associated with the electrostatic ion acoustic wave

$$n=(\chi_e k^2/4\pi e)(\phi+\phi_p), \tag{18}$$

where

$$\chi_e=-\omega_p^2/\{i\nu\omega+\omega^2-\omega(\underline{k}.\underline{V}_a)\} \tag{19}$$

is the electronic susceptibility and

$$\phi_p=-\frac{e}{4m_0\omega_0}\left(\frac{\underline{E}_0.\underline{E}_1}{\omega_1}-\frac{\underline{E}_0^*.\underline{E}_2}{\omega_2}\right) \tag{20}$$

is the ponderomotive potential (Liu and Kaw 1976). In deriving eq. (18) we have neglected the pressure gradient term as

$$\nu > kv_e \text{ and } \omega_0 > k_0 v_e$$

Using tha linear density perturbation $n^L=\chi_e k^2\phi.4\pi e$ for the low-frequency mode and taking the nonlinearity in the current densities at the side-bands through $n^L\underline{V}$ terms we obtain the nonlinear current densities at the high frequency side-bands as

$$\underline{J}_1=-n_0^0 e\underline{V}_1-n^L e\underline{V}_0^*/2 \tag{21}$$

and

$$\underline{J}_2=-n_0^0 e\underline{V}_2-n^L e\underline{V}_0/2 \tag{22}$$

Now, using eq. (18) in the Poisson's equation and eqs. (21) and (22) in the wave equation for the high frequency side-bands we obtain

$$\epsilon\phi=-\chi_e\phi_p, \tag{23}$$

$$\underline{\underline{D}}_1\cdot\underline{E}_1=\frac{4\pi i\omega_1}{c^2}\underline{J}_1^{NL}, \tag{24}$$

$$\underline{\underline{D}}_2\cdot\underline{E}_2=\frac{4\pi i\omega_2}{c^2}\underline{J}_2^{NL}, \tag{25}$$

where

$$\underline{\underline{D}}_{1,2}=(k_{1,2}^2-\omega_{1,2}^2\epsilon_{1,2}/c^2)\underline{\underline{I}}-\underline{k}_{1,2}\underline{k}_{1,2}, \tag{26}$$

$$\epsilon_{1,2}=\epsilon_L-\omega_p^2/\omega_{1,2}^2, \tag{27}$$

and

$$\frac{\omega_p^2}{\epsilon_L(i\nu\omega+\omega^2)}-\frac{K^2k^2c_s^2}{\omega^2-k^2c_s^2}, \tag{28}$$

is the low frequency linear dielectric function, and $\underline{\underline{I}}$ is the unit dyadic. The last term in eq. (28) is the contribution from the lattice. The value of K^2, the electro-mechanical coefficient, for n-GaAs is $\sim 10^{-3}$.

Eliminating ϕ, $\underline{E}_1$ and $\underline{E}_2$ from eqs. (23), (24) and (25) we obtain the following nonlinear dispersion relation for the low-frequency electrostatic ion acoustic wave

$$\epsilon = \frac{\mu_1}{|\underline{\underline{D}}_1|} + \frac{\mu_2}{|\underline{\underline{D}}_2|}, \tag{29}$$

where

$$\mu_1 = \frac{|V_0|^2 \omega_p^4 k^2}{8c^2 \nu^2 \omega^2} \left\{ k_1^2 - \frac{\omega_1^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_1^2}\right)\right\} \left\{ k_{1x}^2 - \frac{\omega_1^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_1^2}\right)\right\}, \tag{30}$$

$$\mu_2 = \frac{|V_0|^2 \omega_p^4 k^2}{8c^2 \nu^2 \omega^2} \left\{ k_2^2 - \frac{\omega_2^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_2^2}\right)\right\} \left\{ k_{2x}^2 - \frac{\omega_2^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_2^2}\right)\right\}, \tag{31}$$

$$|\underline{\underline{D}}_{1,2}| = -\frac{\omega_{1,2}^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_{1,2}^2}\right) \left\{ k_{1,2}^2 - \frac{\omega_{1,2}^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_{1,2}^2}\right)\right\}^2, \tag{32}$$

and

$$|V_0| = eE_0/m_0\omega_0$$

In deriving the dispersion relation, eq. (29) we assume that $\underline{k}_1$ and $\underline{k}_2$ lie in the XZ-plane.

3. Growth rate

We now apply the resonance conditions

$$\begin{aligned} \omega_{1,2} &= \omega \mp \omega_0, \\ \underline{k}_{1,2} &= \underline{k} \mp \underline{k}_0, \end{aligned} \tag{33}$$

where

$$k_{0,1,2} = \frac{\omega_{0,1,2}}{c}\left(\epsilon_L - \frac{\omega_p^2}{\omega_{0,1,2}^2}\right)^{1/2} \tag{34}$$

For simplicity we consider the case where both the high frequency side-bands propagate in the same direction in the XZ-plane making an angle θ with the direction of incidence of the incident laser beam. This consideration does not change the order of the growth rate of the ion acoustic mode.

Now, the phase matching condition that the group velocity of the incident laser radiation will be equal to the phase velocity of the low frequency mode demands that the wavelength of the low-frequency ion acoustic mode must be much greater than the wavelength of the incident laser radiation.

In the absence of the incident pump wave the coupling coefficients vanish, $\mu_1 = \mu_2 = 0$. However, in the presence of the pump wave, the complex frequency is modified. Near the resonance conditions we can have the following expansions (Liu and Kaw, 1976)

$$\begin{aligned} \omega &= \omega_r + i\gamma, \\ \epsilon &= \epsilon_r(\omega_r, \underline{k}) + i\epsilon_i(\omega, \underline{k}) \\ &\simeq \epsilon_r(\omega_r, \underline{k}) + i\gamma \frac{\partial \epsilon_r}{\partial \omega} + i\epsilon_i(\omega_r, \underline{k}), \end{aligned} \tag{35}$$

where ω_r is the root of $\epsilon_r(\omega_r, \underline{k}) = 0$. The linear damping rate of the electrostatic ion acoustic wave is defined as

$$\gamma_L = \epsilon_i / (\partial \epsilon_r / \partial \omega_r) \tag{36}$$

Therefore, we can write

$$\epsilon(\omega, \underline{k}) \simeq i(\gamma + \gamma_L) \frac{\partial \epsilon_r}{\partial \omega} \tag{37}$$

Similarly,

$$|\underline{\underline{D}}_{1,2}| \simeq i(\gamma + \gamma_{L1,L2}) \frac{\partial |\underline{\underline{D}}_{1,2}|}{\partial \omega_{1,2}}, \tag{38}$$

where γ_{L1} and γ_{L2} are the linear damping rates of the high-frequency side-bands. To a good approximation we can

$$\begin{aligned} \gamma_L &= \epsilon_L \nu^3 k^2 C_s^2 K^2 / 2\omega^2 \omega_p^2, \\ \gamma_{L1} &= \gamma_{L2} \simeq 0 \end{aligned} \tag{39}$$

Hence, the growth rate of the low frequency ion acoustic wave in n-GaAs in the presence of the incident laser radiation is given by

$$\begin{aligned} (\gamma + \gamma_L)\gamma &\simeq \gamma_0^2 \\ &= -\frac{|V_0|^2 \omega_p^4 k^2}{8c^2 \nu^2 \omega^2 (\partial \epsilon_r / \partial \omega)} \\ &\left[\frac{\left\{ k_{1x}^2 - \frac{\omega_1^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_1^2}\right) \right\}}{\left\{ -\frac{2\omega_1}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_1^2}\right) \right\} \left\{ k_1^2 - \frac{3\omega_1^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_1^2}\right) \right\}} \right. \\ &\left. + \frac{\left\{ k_{2x}^2 - \frac{\omega_2^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_2^2}\right) \right\}}{\left\{ -\frac{2\omega_2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_2^2}\right) \right\} \left\{ k_2^2 - \frac{3\omega_2^2}{c^2}\left(\epsilon_L - \frac{\omega_p^2}{\omega_2^2}\right) \right\}} \right] \\ &\simeq \frac{|V_0| C_s|^2 \epsilon_L^2 \nu^2 \omega^2 K^2 \cos^2\theta}{16 c^2 k_0^2}, \end{aligned} \tag{40}$$

where V_0 is the growth rate in absence of the linear damping.

In the presence of the linear damping of the ion acoustic wave, the growth rate is given by

$$\gamma=\{(\gamma_L^2+4\gamma_0^2)^{1/2}-\gamma_L\}/2 \tag{41}$$

To have some numerical appreciation of the results we calculate the gro$\wedge$th rates for the following parameters of n-GaAs :

$\epsilon_L=13.5$, $T_e=300°K$, $\nu\simeq 2\times 10^{11}$ rad.sec^{-1},

$\omega_p-0.9\omega_0$, $\theta=30°$ and $\omega_0=1.778\times 10^{14}$ rad.sec^{-1}

(corresponding to a CO_2 laser).

When the drift velocity of electrons, $\underline{V}_0$ is comparable to the velocity of the ion acoustic wave (C_s $-\omega/k$), the growth rate of the modulation instability turns out to be $\sim 10^4$ rad.sec^{-1} at $k\simeq 10^3$ cm^{-1}.

It may be mentioned here that since $\underline{E}_d \parallel \hat{x}$ and the Z-component of the ponderomotive force is taken into account, only the ponderomotive nonlinearity appears in our analysis and the mass nonlinearity does not come into play at all. However, one can take into account the effect of the nonlinearity due to the field dependent effective mass of electrons if the dc electric field is applied in the direction of propagation of the incident wave.

4. Discussion

A high power laser radiation propagating through a piezoelectric semiconductor, viz., n-type GaAs gets modulated by a low frequency ion acoustic perturbation propagating in the direction of the incident laser radiation. The ponderomotive nonlinearity dominates over the other possible nonlinear sources, viz., the field-dependent effective mass and collision frequency of electrons. For typical plasma parameters in an n-type GaAs sample the growth rate of the modulation instability is $\sim 10^4$ rad.sec^{-1}. The results of this paper suggest also that the process of parametric instability can be easily verified in semiconductors where the plasma parameters can be varied over a wide range of values without much difficulty.

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